

MATHEMATICS MODULE ONE

INTRODUCTION TO THE LTF GUIDES THROUGH ANALYSIS OF PIECEWISE FUNCTIONS

Description

Middle school and high school Pre-AP teachers attend separate sessions where they examine an overview of the Laying the Foundation mathematics program which is focused on making connections between prior and future learning. In the training, teachers investigate student lessons that describe and analyze piecewise functions and demonstrate how AP concepts are developed from sixth grade through pre-calculus. The day concludes with a tour of the LTF website and a review of the many available resources. Participants are given passwords to access the protected materials on the LTF website, including lessons, quizzes, and free response questions with student samples.

Learner Outcomes

Teachers will:

- ▶ Compare expectations of students from sixth grade math through pre-calculus on the topic of analysis of piecewise functions.
- ▶ Identify multiple representations (physical, verbal, analytical, numerical, and graphical) in student lessons on analysis of piecewise functions.
- ▶ Acquire deeper content knowledge by investigating lessons and assessments.
 - Interpret distance-time and rate-time graphs
 - Role play to model distance-time and rate-time graphs
 - Write piecewise functions using a transformational approach.
 - Analyze piecewise functions
- ▶ Identify vocabulary that is important in teaching analysis of functions
- ▶ Identify strategies to enhance students' understanding.
- ▶ Demonstrate an understanding of graphing calculator skills used in analyzing functions.

Lessons and Standards

The student lessons and activities included in this module's training address the following Standards for Mathematical Practice.

Standards for Mathematical Practice		Specific Standards
1	Make sense of problems and persevere in solving them.	<i>Specific standards for middle and high school are included on each lesson.</i>
2	Reason abstractly and quantitatively.	
3	Construct viable arguments and critique the reasoning of others.	
4	Model with mathematics.	
5	Use appropriate tools strategically.	
6	Attend to precision.	
7	Look for and make use of structure.	
8	Look for and express regularity in repeated reasoning.	

MATHEMATICS MODULE TWO AREAS AND VOLUMES

Description

Middle school and high school Pre-AP teachers attend separate sessions where they explore manipulative-rich student lessons that investigate the area of two-dimensional figures, as well as surface area and volume of three-dimensional solids that result from revolving the planar figures about an axis. As the lessons progress through the vertical strand, teachers learn how students graph the original planar figure by first plotting points, then graphing equations, and finally graphing systems of inequalities. Training will emphasize the connections from sixth grade through pre-calculus.

Learner Outcomes

Teachers will:

- ▶ Compare expectations of students from sixth grade math through pre-calculus on the topic of areas and volumes.
- ▶ Acquire deeper content knowledge by investigating lessons and assessments.
 - Identify the relationship among area formulas and express formulas as a function of specific variables.
 - Apply multiple methods of calculating area for regions in the coordinate plane.
 - Model and visualize 3-dimensional solids formed when various regions are revolved about horizontal or vertical lines.
 - Calculate the surface area and volume of the solids of revolution.
- ▶ Identify vocabulary that is important in teaching areas and volumes.
- ▶ Identify strategies to enhance students' understanding of areas and volumes.

Lessons and Standards

The student lessons and activities included in this module's training address the following Standards for Mathematical Practice.

Standards for Mathematical Practice		Specific Standards
1	Make sense of problems and persevere in solving them.	<i>Specific standards for middle and high school are included on each lesson.</i>
2	Reason abstractly and quantitatively.	
3	Construct viable arguments and critique the reasoning of others.	
4	Model with mathematics.	
5	Use appropriate tools strategically.	
6	Attend to precision.	
7	Look for and make use of structure.	
8	Look for and express regularity in repeated reasoning.	

MATHEMATICS MODULE THREE

RATE OF CHANGE: AVERAGE AND INSTANTANEOUS

Description

Middle school and high school Pre-AP teachers attend separate sessions where they explore student lessons connecting slope to the AP Calculus concept of rate of change. They will explore lessons that differentiate between the average and the instantaneous rate of change of a function. Middle school teachers will explore manipulative-rich lessons that introduce the concepts of constant rate of change and average rate of change. Additional lessons introduce high school teachers to the concept of a curve with a varying slope and to the calculus notation for a derivative to represent that slope. High school teachers also learn strategies for using a graphing utility in parametric mode to graph a curve that is not a function. Training will emphasize the connections from sixth grade through pre-calculus.

Learner Outcomes

Teachers will:

- ▶ Compare expectations of students from sixth grade math through pre-calculus on the topic of rate of change.
- ▶ Acquire content knowledge by investigating lessons and assessments.
 - Differentiate between the average and the instantaneous rate of change of a function.
 - Calculate average rate of change and estimate instantaneous rate of change.
 - Model rate of change using exploratory activities, role play, and CBR's.
- ▶ Identify vocabulary and notation that is important in teaching rate of change.
- ▶ Identify strategies to enhance students' understanding of rate of change.

Lessons and Standards

The student lessons and activities included in this module's training address the following Standards for Mathematical Practice.

Standards for Mathematical Practice		Specific Standards
1	Make sense of problems and persevere in solving them.	<i>Specific standards for middle and high school are included on each lesson.</i>
2	Reason abstractly and quantitatively.	
3	Construct viable arguments and critique the reasoning of others.	
4	Model with mathematics.	
5	Use appropriate tools strategically.	
6	Attend to precision.	
7	Look for and make use of structure.	
8	Look for and express regularity in repeated reasoning.	

MATHEMATICS MODULE FOUR

GRAPHICAL DISPLAYS; DISTRIBUTIONS: MEASURES OF CENTER, VARIABILITY, AND SHAPE

Description

Middle school and high school Pre-AP teachers attend separate sessions where they explore the concept of graphical displays by working student lessons that construct, compare, analyze, and interpret box-and-whisker plots, line plots (dot plots), and stem-and-leaf plots. Each lesson employs real-world data to construct, by hand and with a graphing calculator, appropriate graphical displays and to analyze the graph using measures of central tendency, variability, and shape. The training concludes with an exploration of AP Statistics free-response questions that are accessible to Pre-AP students. Training will emphasize the connections from sixth grade through pre-calculus.

Learner Outcomes

Teachers will:

- ▶ Compare expectations of students from sixth grade math through pre-calculus on the topic of graphical displays and distributions.
- ▶ Acquire content knowledge by investigating lessons and assessments.
 - Construct, compare, analyze, and interpret bar graphs, histograms, box-and-whisker plots, line plots (dot plots), and stem-and-leaf plots
 - Analyze graphs using measures of central tendency, variability, and shape.
 - Model graphical displays using role play
- ▶ Identify vocabulary that is important in teaching accumulation.
- ▶ Identify strategies to enhance students' understanding of graphical displays and distributions.
- ▶ Demonstrate an understanding of graphing calculator skills used in creating graphical displays.

Lessons and Standards

The student lessons and activities included in this module's training address the following Standards for Mathematical Practice.

Standards for Mathematical Practice		Specific Standards
1	Make sense of problems and persevere in solving them.	<i>Specific standards for middle and high school are included on each lesson.</i>
2	Reason abstractly and quantitatively.	
3	Construct viable arguments and critique the reasoning of others.	
4	Model with mathematics.	
5	Use appropriate tools strategically.	
6	Attend to precision.	
7	Look for and make use of structure.	
8	Look for and express regularity in repeated reasoning.	

MATHEMATICS MODULE FIVE ACCUMULATION

Description

Middle school and high school Pre-AP teachers attend separate sessions where they explore the concept of accumulating area that leads to the concept of the definite integral in AP Calculus. Teachers will explore techniques for approximating area of various closed regions through manipulative-rich middle school lessons. High school lessons extend these techniques to determining the area under a curve using geometric figures. Training will emphasize the connections from sixth grade through pre-calculus.

Learner Outcomes

Teachers will:

- ▶ Compare expectations of students from sixth grade math through pre-calculus on the topic of accumulation.
- ▶ Acquire content knowledge by investigating lessons and assessments.
 - Estimate areas of irregular figures and recognize that smaller subdivisions lead to accurate estimates.
 - Model the surface area of a sphere and develop its formula.
 - Estimate the area between a curve and the x -axis using left-hand, right-hand, and midpoint rectangles.
 - Estimate the area between a curve and the x -axis using trapezoids and connect the trapezoidal approximation to the mean of the left-hand and right-hand rectangle approximations
- ▶ Identify vocabulary and notation that is important in teaching accumulation.
- ▶ Identify strategies to enhance students' understanding of accumulation.
- ▶ Demonstrate an understanding of graphing calculator skills used in accumulation.

Lessons and Standards

The student lessons and activities included in this module's training address the following Standards for Mathematical Practice.

Standards for Mathematical Practice		Specific Standards
1	Make sense of problems and persevere in solving them.	<i>Specific standards for middle and high school are included on each lesson.</i>
2	Reason abstractly and quantitatively.	
3	Construct viable arguments and critique the reasoning of others.	
4	Model with mathematics.	
5	Use appropriate tools strategically.	
6	Attend to precision.	
7	Look for and make use of structure.	
8	Look for and express regularity in repeated reasoning.	

MATHEMATICS MODULE SIX PROBABILITY

Description

Middle school and high school Pre-AP teachers attend separate sessions where teachers will delve into student lessons that investigate probability. Techniques include using a sample space, conducting simulations, and collecting data. Teachers will discover and apply Pascal's Triangle and the Binomial Theorem to probability. Additional topics include geometric probability and permutations and combinations. Training will emphasize the connections from sixth grade through pre-calculus.

Learner Outcomes

Teachers will:

- ▶ Compare expectations of students from sixth grade math through pre-calculus on the topic of probability.
- ▶ Acquire content knowledge by investigating lessons and assessments.
 - Use Venn diagrams, tree diagrams, and sample spaces.
 - Differentiate between experimental and theoretical probability.
 - Use simulations and data collection to explore probability questions.
 - Apply Pascal's Triangle and the Binomial Theorem to probability.
 - Calculate geometric probability.
 - Use permutations and combinations.
- ▶ Identify vocabulary and notation that is important in teaching probability.
- ▶ Identify strategies to enhance students' understanding of probability.
- ▶ Demonstrate an understanding of graphing calculator skills used in probability.

Lessons and Standards

The student lessons and activities included in this module's training address the following Standards for Mathematical Practice.

Standards for Mathematical Practice		Specific Standards
1	Make sense of problems and persevere in solving them.	<i>Specific standards for middle and high school are included on each lesson.</i>
2	Reason abstractly and quantitatively.	
3	Construct viable arguments and critique the reasoning of others.	
4	Model with mathematics.	
5	Use appropriate tools strategically.	
6	Attend to precision.	
7	Look for and make use of structure.	
8	Look for and express regularity in repeated reasoning.	

MATHEMATICS MODULE SEVEN

POSITION/VELOCITY/ACCELERATION

Description

Middle school and high school Pre-AP teachers attend separate sessions where teachers will explore the concepts and relationships of position, velocity, and acceleration. Participants will use physical activities and technology such as a CBR and a graphing calculator to more fully understand the concepts. Lessons include sketching a graph from a story, interpreting graphs from a verbal description, and analyzing and comparing graphs of position, velocity, and acceleration. Training will emphasize the connections from sixth grade through pre-calculus.

Learner Outcomes

Teachers will:

- ▶ Compare expectations of students from sixth grade math through pre-calculus on the topic of position/velocity/acceleration.
- ▶ Acquire content knowledge by investigating lessons and assessments.
 - Create position/velocity/acceleration graphs using exploratory activities, role play, and CBR's.
 - Use verbal descriptions to identify transformational changes in a position graph.
 - Analyze distance and speed graphs.
 - Understand the relationship between position, velocity, and acceleration graphs
 - Differentiate between velocity and speed.
 - Interpret the motion of a particle along a horizontal line.
- ▶ Identify vocabulary for teaching position/velocity/acceleration.
- ▶ Identify strategies to enhance students' understanding of probability.
- ▶ Demonstrate an understanding of graphing calculator skills used in position/velocity/acceleration.

Lessons and Standards

The student lessons and activities included in this module's training address the following Standards for Mathematical Practice.

Standards for Mathematical Practice		Specific Standards
1	Make sense of problems and persevere in solving them.	<i>Specific standards for middle and high school are included on each lesson.</i>
2	Reason abstractly and quantitatively.	
3	Construct viable arguments and critique the reasoning of others.	
4	Model with mathematics.	
5	Use appropriate tools strategically.	
6	Attend to precision.	
7	Look for and make use of structure.	
8	Look for and express regularity in repeated reasoning.	

MATHEMATICS MODULE EIGHT

LIMITS

Description

Middle school and high school Pre-AP teachers attend separate sessions where teachers will explore the concept of limits from various perspectives. Student lessons use pattern recognition, perimeter and area of polygons, secant and tangent lines to circles and ellipses, and end-behavior of rational functions to lead to an informal notion of a limit. Training will emphasize the connections from sixth grade through pre-calculus.

Learner Outcomes

Teachers will:

- ▶ Compare expectations of students from sixth grade math through pre-calculus on the topic of limits.
- ▶ Acquire content knowledge by investigating lessons and assessments.
 - Role play to investigate limits.
 - Use pattern recognition, exponential growth and decay, Fibonacci-like sequences, perimeter and area of polygons, secant and tangent lines to circles and ellipses, and end-behavior of rational functions to develop an informal notion of a limit.
 - Analyze limits from a numeric, geometric, and algebraic perspective.
 - Differentiate between a calculated value and a limiting value.
- ▶ Identify vocabulary and notation that is important in teaching limits.
- ▶ Identify strategies to enhance students' understanding of limits.
- ▶ Demonstrate an understanding of graphing calculator skills used in limits.

Lessons and Standards

The student lessons and activities included in this module's training address the following Standards for Mathematical Practice.

Standards for Mathematical Practice		Specific Standards
1	Make sense of problems and persevere in solving them.	<i>Specific standards for middle and high school are included on each lesson.</i>
2	Reason abstractly and quantitatively.	
3	Construct viable arguments and critique the reasoning of others.	
4	Model with mathematics.	
5	Use appropriate tools strategically.	
6	Attend to precision.	
7	Look for and make use of structure.	
8	Look for and express regularity in repeated reasoning.	

MATHEMATICS MODULE NINE

OPTIMIZATION: AREA AND VOLUME APPLICATIONS

Description

Middle school and high school Pre-AP teachers attend separate sessions where they explore manipulative-rich student lessons that investigate the concept of optimization. Lessons will include maximizing and minimizing area and volume to determine an optimum solution. Training will emphasize the connections from sixth grade through pre-calculus.

Learner Outcomes

Teachers will:

- ▶ Compare expectations of students from sixth grade math through pre-calculus on the topic of optimization.
- ▶ Acquire content knowledge by investigating lessons and assessments.
 - Use measurement and models to investigate optimization.
 - Write functions for area and volume from descriptions of real-world applications, determine reasonable domains and ranges, and then determine optimum solutions.
- ▶ Identify vocabulary that is important in teaching optimization.
- ▶ Identify strategies to enhance students' understanding of optimization.
- ▶ Demonstrate an understanding of graphing calculator skills used in optimization.

Lessons and Standards

The student lessons and activities included in this module's training address the following Standards for Mathematical Practice.

Standards for Mathematical Practice		Specific Standards
1	Make sense of problems and persevere in solving them.	<i>Specific standards for middle and high school are included on each lesson.</i>
2	Reason abstractly and quantitatively.	
3	Construct viable arguments and critique the reasoning of others.	
4	Model with mathematics.	
5	Use appropriate tools strategically.	
6	Attend to precision.	
7	Look for and make use of structure.	
8	Look for and express regularity in repeated reasoning.	

MATHEMATICS MODULE TEN

LINEAR AND NON-LINEAR BIVARIATE DATA

Description

Middle school and high school Pre-AP teachers attend separate sessions where they explore manipulative-rich student lessons that investigate the concept of linear and non-linear bivariate data. Teachers will investigate data coding, fit functions to data, use models to predict values, and determine residuals. In addition, slope and intercepts will be explored in the context of the questions. Training will emphasize the connections from sixth grade through pre-calculus.

Learner Outcomes

Teachers will:

- ▶ Compare expectations of students from sixth grade math through pre-calculus on the topic of analyzing bivariate data.
- ▶ Acquire content knowledge by investigating lessons and assessments.
 - Collect and use real-world data to develop the skills of coding and graphing data, fitting functions to data, using the models to predict other values, and interpreting the meanings of slope and intercepts in the context of the situation.
 - Analyze regression models using residuals.
 - Use techniques for straightening curved data to identify models of best fit.
- ▶ Identify vocabulary that is important in teaching bivariate data applications.
- ▶ Identify strategies to enhance students' understanding of analyzing bivariate data.
- ▶ Demonstrate an understanding of graphing calculator skills used in analyzing bivariate data.

Lessons and Standards

The student lessons and activities included in this module's training address the following Standards for Mathematical Practice.

Standards for Mathematical Practice		Specific Standards
1	Make sense of problems and persevere in solving them.	<i>Specific standards for middle and high school are included on each lesson.</i>
2	Reason abstractly and quantitatively.	
3	Construct viable arguments and critique the reasoning of others.	
4	Model with mathematics.	
5	Use appropriate tools strategically.	
6	Attend to precision.	
7	Look for and make use of structure.	
8	Look for and express regularity in repeated reasoning.	

MATHEMATICS MODULE ELEVEN
ANALYSIS OF FUNCTIONS: TRANSFORMATION

Description

Middle school and high school Pre-AP teachers attend separate sessions where teachers will delve into student lessons that investigate transformations and parent functions. They will explore translations, reflections, rotations, and dilations analytically, graphically, and numerically. Training will emphasize the connections from sixth grade through pre-calculus.

Learner Outcomes

Teachers will:

- ▶ Compare expectations of students from sixth grade math through pre-calculus on the topic of transformations.
- ▶ Acquire content knowledge by investigating lessons and assessments.
 - Identify parent functions.
 - Examine the effects of transformations physically, verbally, analytically, numerically, and graphically.
 - Interpret the meaning of transformations in the context of a real-world situation.
- ▶ Identify vocabulary that is important in teaching transformations.
- ▶ Identify strategies to enhance students' understanding of transformations.
- ▶ Demonstrate an understanding of graphing calculator skills used in transformations.

Lessons and Standards

The student lessons and activities included in this module's training address the following Standards for Mathematical Practice.

Standards for Mathematical Practice		Specific Standards
1	Make sense of problems and persevere in solving them.	<i>Specific standards for middle and high school are included on each lesson.</i>
2	Reason abstractly and quantitatively.	
3	Construct viable arguments and critique the reasoning of others.	
4	Model with mathematics.	
5	Use appropriate tools strategically.	
6	Attend to precision.	
7	Look for and make use of structure.	
8	Look for and express regularity in repeated reasoning.	

MATHEMATICS MODULE TWELVE

Rate of Change: Related Rates

Description

Middle school and high school Pre-AP teachers attend separate sessions where they explore manipulative-rich student lessons that investigate the concept of related rates. The session will begin with a review of literal equations and how these equations can be emphasized from sixth grade through pre-calculus. Teachers will explore dynamic situations where a change in one quantity results in a change in another quantity through related rates applications involving triangles, curves, areas, and volumes.

Learner Outcomes

Teachers will:

- ▶ Compare expectations of students from sixth grade math through pre-calculus on the topic of related rates.
- ▶ Acquire content knowledge by investigating lessons and assessments.
 - Solve literal equations.
 - Model dynamic situations where a change in one quantity causes a related change in another quantity, and the rates at which those quantities change are linked through their mathematical relationship
 - Solve real-life applications questions for related rates situations.
- ▶ Identify vocabulary that is important in teaching related rates.
- ▶ Identify strategies to enhance students' understanding of related rates.

Lessons and Standards

The student lessons and activities included in this module's training address the following Standards for Mathematical Practice.

Standards for Mathematical Practice		Specific Standards
1	Make sense of problems and persevere in solving them.	<i>Specific standards for middle and high school are included on each lesson.</i>
2	Reason abstractly and quantitatively.	
3	Construct viable arguments and critique the reasoning of others.	
4	Model with mathematics.	
5	Use appropriate tools strategically.	
6	Attend to precision.	
7	Look for and make use of structure.	
8	Look for and express regularity in repeated reasoning.	